

SUPPLEMENTARY TEXT

Here are the detailed equation derivations to introduce how to calculate knee moments based on the proposed human dynamic model.

The coordinates of each COM are as follows:

$$\begin{cases} \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} L_b \sin \theta_{Tr} - L_{th} \sin \theta_{Th} + L_{sh} \sin \theta_{Sh} \\ L_{sh} \cos \theta_{Sh} + L_{th} \cos \theta_{Th} + L_b \cos \theta_{Tr} \end{bmatrix} \\ \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} L_{sh} \sin \theta_{Sh} - \frac{1}{2} L_{th} \sin \theta_{Th} \\ L_{sh} \cos \theta_{Sh} + \frac{1}{2} L_{th} \cos \theta_{Th} \end{bmatrix} \\ \begin{bmatrix} x_3 \\ y_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} L_{sh} \sin \theta_{Sh} \\ \frac{1}{2} L_{sh} \cos \theta_{Sh} \end{bmatrix} \end{cases} \quad (7)$$

The square of the velocity of each COM is:

$$v_i^2 = \dot{x}_i^2 + \dot{y}_i^2 \quad (8)$$

The relative kinetic energies are T_1 , T_2 , and T_3 respectively; the potential energies are U_1 , U_2 , and U_3 respectively. The total kinetic energy T and the total potential energy U of the system are the sums of these individual contributions.

$$\begin{cases} T_1 = \frac{1}{2} M_b v_1^2 + \frac{1}{2} I_1 \dot{\theta}_{Tr}^2 \\ T_2 = \frac{1}{2} M_{th} v_2^2 + \frac{1}{2} I_2 \dot{\theta}_{Th}^2 \\ T_3 = \frac{1}{2} M_{sh} v_3^2 + \frac{1}{2} I_3 \dot{\theta}_{Sh}^2 \end{cases} \quad (9)$$

$$\begin{cases} U_1 = M_b g (L_{sh} \cos \theta_{Sh} + L_{th} \cos \theta_{Th} + L_b \cos \theta_{Tr}) \\ U_2 = M_{th} g \left(L_{sh} \cos \theta_{Sh} + \frac{1}{2} L_{th} \cos \theta_{Th} \right) \\ U_3 = \frac{1}{2} M_{sh} g L_{sh} \cos \theta_{sh} \end{cases} \quad (10)$$

where the moments of inertia for the centroids at AB, BC, and CD are I_1 , I_2 , and I_3 , respectively.

$$I_1 = \frac{M_b L_b^2}{12}, I_2 = \frac{M_{th} L_{th}^2}{12}, I_3 = \frac{M_{sh} L_{sh}^2}{12} \quad (11)$$

The Lagrangian function of this model is:

$$L = T(\theta, \dot{\theta}) - U(\theta) \quad (12)$$

The knee torque (τ_K) can be derived from (8):

$$\tau_K = \frac{d}{dt} \frac{\partial L}{\partial \dot{\theta}_K} - \frac{\partial L}{\partial \theta_K} \quad (13)$$

where $\theta_K = \theta_{Th} + \theta_{Sh}$ represents the knee angle. The parameters $M_b, M_{th}, M_{sh}, L_b, L_{th}$ and L_{sh} are calculated using data in [1].

[1] Armstrong, H. G., 1988, "Anthropometry and mass distribution for human analogues," Military Male Aviators, 1.